A study of fatigue life estimation for differential reducers

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Abstract

The expected lives of reducers as well as its performances are frequently requested in many industrial products. This paper presents the methods of fatigue life estimation for a differential reducer (DR) which possesses the properties, large speed-reduction ratios, high rigidness and compact structure. The DR is designed based on the principles of differential displacements of the deceleration gear rings. The geometric models of the mechanism are constructed for interference checking and structural stress analysis. The stresses of tooth root bending and surface contacting of the gear rings are studied for evaluating the fatigue lives of the DR under various loadings. Finite element methods (FEM) are used as a tool to analyze the stresses of the gear rings. The analyzed results are compared with formula calculations to identify the accuracy of analyzing also to modifying the setting of the related parameters. The fatigue life estimation is implemented according to the analyzed stresses in cooperation with the stress-life cycle curves reported in AGMA Standard. The studied results show that the bending fatigue failures would be prior to the contacting fatigue failures. The studies provide some useful information including the allowed loadings, fatigue lives and reliabilities of the DR in using.

Key words: fatigue life, differential reducer, reliability.

1. Introduction

Reducers are applied in many devices for power transmission and speed reduction such as machine tools, enterprise machinery and industrial robots, etc. The reducers applied in robots must possess the properties, high speed-reduction ratios, large torque output and compact structure, so that it was extensively studied for satisfying the function requirements[1]. In this study, a reducer designed based on differential displacement theories named as differential reducer (RD), is proposed including the methods of structural stress analysis and fatigue life prediction. The geometric models of the design are constructed by SolidWorks software for interference check and motion simulation. The induced stresses of the key components under various loadings are analyzed using finite element methods (FEM) for observing the fractured spots. The analyzed results are also compared with the stresses obtained by formula calculation for identify the accuracy of analyzing. The analyzed results are further applied to predict the fatigue lives of the DR in using.

The gear rings play a critical role in speed-reduction ratios so that it is the key components in the DR. The designing of the gear rings directly effects the strength and the speed-reduction ratios of the DR and its dynamic properties. The challenges of designing are the profiles of the tooth shapes to meet the needs in strength, precisions and expected lives, meanwhile, small noise and vibration during operating. The noise and vibration emissions are the major problems of gear transmission as well as its strength. The calculations of gear designing are normally performed according to the handbook formulas which were often implemented by simple computer programs.

Focusing on gear damages, the failure types can be divided into 5 categories according to the definitions in ANSI/AGMA[2]. They are (1) wear, (2) plastic flow, (3) breakage, (4) contact fatigue and (5) cracking. One type of hardening crack is the internal stress rupture reported by Alban[3]. Internal stress rupture is a direct result of the accumulated residual tensile stresses exceeding the strength of the core material in the gear tooth. Different types of failure usually are treated either by heat processing or by design improvements [4]. The designer often faces a trade-off between different function qualities, such as contact fatigue resistance, bending fatigue strength and noise emissions. For example, Thompson et al.[5] made a trade-off analysis of minimum volume design for multi-stage spur gears. Ciavarella and Demelo[6] presented the numerical methods of
optimization for stress concentration and fatigue life. On the other hand, the increasing of strengths and working speeds in current applications has leaded a requirement for better specifications on fatigue behaviors. If the crack is not immediately obvious, it is normally detected after a short period of service life.

To predict the expected lives of the DR, the procedures of evaluating the fracturing stresses and fatigue lives of the gear rings must be given so that the loading capacity can be determined as well as the unexpected failures can be avoid. The general methods for fatigue testing and analysis were reported in Ref.[7]. The fatigue cracks of the gear rings including contact fatigue and tooth root bending fatigue are initially to occur at the surface. It is a result of cumulated damage of materials caused by the action of repeated or fluctuating stresses. Fatigue testing of materials is often carried out at various stress levels to define the magnitude of fatigue limit reported as a specific number of cycles. Commonly, the fatigue life curve of materials is always expressed with a regression line. The lives at each stress level should be a distribution because of the scatter of the life data.

The fatigue stresses of the gear rings must be determined before evaluating its fatigue lives. The stress estimation methods including root bending stress (RBS) and surface contacting stresses (SCS) have been reported in international standards ISO 6336[ 8] and AGMA standards[ 9]. The existed problems are the evaluated formulas involving many unknown parameters which is uneasy to be given accurately because they are varying depending upon material properties, tooth shape design and operational conditions. To treat the problems of stress parameters uneasily decided, Finite Element Analysis (FEA) tool, ANSYS, is used to determine the stresses of the gear ring since it can provide satisfactory solutions of stress analysis. An application of FEA in fatigue life estimation for dental implants was reported in Ref[ 10].

In this paper, ANSYS is use as a tool to analyze the RBS and the SCS of the gear rings. The analyzed results are further substituted into the life estimation model to predict the expected life. The life estimation models including bending fatigue and contacting fatigue are developed based on the S-N curve of materials. The graphs provides the corresponding fatigue strength for steel reported at a specific number of stress cycles. The stress cycle curves for pitting resistance and bending strength of steel gears has been suggested in AGMA Standard. The differences of life estimation between the proposed models and the AGMA models are compared to identify the accuracy. Normally, the fatigue lives vary depending upon the load capacities and the tooth’s size. It is useful to consider the fatigue resistance level in case of a high number of stress cycles. The necessity for greater accuracy in the determination of fatigue limit for steel with applications in high speed gear transmission has led to testing and new studies in high stress cycle.

2. Design of Differential Reducer (DR)

A novel design about reducers is proposed to perform high speed-reduction ratios and to satisfy the needs of space limited. The geometric structure is designed based on differential displacements of the gear rings. The motion is simulated for identify the feasibility of the designing.

The DR is designed based on the principles of differential displacements which is similar to the moving of the slid meter to the fixed meter of the traditional caliper. The mechanism mainly includes four parts, the fixed ring, the slid ring, the intermediary ring and the off-centre cam. The combination of the geometric models is illustrated in Figure 1. The intermediary ring is driven by an off-centre cam which is connecting to the power input shaft as well as the slid ring to the power output. The fixed and the slid rings are designed with different teeth so that the differential displacements can be generated when the intermediary ring is rotating circling the center of the fixed ring. The differential movements are arisen when the teeth of the slid ring are forced to align to the fixed ring due to the mating transmission of the gears. The fixed ring is usually fixed on the support box for leading the intermediary ring rotating in the form of planetary motions. The scales of speed reduction ratios are decided depending on the difference of the tooth numbers of the fixed and slid gear rings.
The off-centre cam is installed in the intermediary ring inside and is driven by the power shaft. It drives the intermediary ring to generate planetary motions circling the fixed ring. The off-centre cam can be designed with counterweight at the other side to dispel the imbalance during operating. The intermediary ring is designed with the same tooth forms so that the mating transmission of the three gear rings can be carried out. The bearings of the power shaft can be installed at two sides of the gear box to offer the support of power torques. The tooth numbers of the fixed and slid rings are decided according to the ratios of speed reduction. The tooth sizes on the gear rings are designed from the transmitted torques. Ideally, the slid ring and the sustained bearings can be designed as a whole for shortening the length of the reducer as well as structure compactness.

The geometric structure of the DR is designed using SolidWorks software. The parametric solid models of the gear teeth are generated based on involute full depth profile which the profiles are shifted to avoid interference occurrence during operating. Parametric design means that the physical shape of the teeth is driven by the values assigned to the attributes (primarily dimensions) of its features. The detailed design for the gear rings including the fixed, slid and intermediary rings, in this example, are set to module \( m = 1 \), pressure angle \( \theta = 20^\circ \). The tooth numbers of the fixed, slid and intermediary gear rings are set to \( \{50, 49, 48\} \), respectively. The off-centre cam was designed according to the geometric needs of kinematic transmission. The components are assembled into a whole by setting the coupled conditions of the components. The ratios of speed reduction for the DR are

\[
s = \frac{z_s - z_c}{z_s} = -\frac{1}{49}
\]

where \( z_s, z_c \) stand for the tooth numbers of the slid and the fixed gears, respectively.

3. Stress models

Reviewing the route of power transmission of the DR, the structural weaknesses of the design may occur at the engaged positions of the gear rings. The possible failures of the gear rings are either gear teeth breakage due to over large loadings or the fatigue fractures due to the periodical stresses.

3.1 Bending stress

Bending stress is commonly evaluated based on the Lewis equation. It models a gear tooth taking the full load at its tip as a simple cantilever beam. The Lewis equation is stated as below

\[
\sigma_f = \frac{F_d}{bY}
\]
\[ Y = \frac{2xd}{3}, \quad x = \frac{t^2}{4d} \]

In above equation, \( \sigma_x \) is the maximum bending stress, \( b \) is the face width, \( d \) the reference diameter (diametric pitch), \( Y \) the Lewis form factor, \( F_t \) is the tangential load, \((t, l)\) the height and length of the approximate cantilever beam. The form factor, \( Y \) is a function of the number of teeth, pressure angle, and involutes depth of the gear. The maximum bending stress is decided depending on the variable \( x \) which considers the geometry of the tooth, but does not include stress concentration.

To analyze the worst load condition on gear teeth, the tip load condition as proposed originally by Lewis is not the most critical. In nearly all gear designs, the contact ratio is high enough to put a second pair of teeth in contact when one pair has reached the tip-load condition on one member. Hence, considering worst load condition in this work, the Lewis factor, \( Y \), for a gear with 20 teeth, full depth profile, and 20 degree pressure angle is 0.33.

On the other hand, the ISO published standards ISO 6336-3\[ 8\]also reported the methods of calculating RBS (Root Bending Stress) for a pair of spur gears and helical gears. It is defined as

\[ \sigma_v = \frac{F_t}{b m_n} Y_F Y_Y K_A K_F K_{F_0} K_{F_{\beta}} \]  

The formula involves a lot of unknown parameters such as nominal tangential load (N), face width and normal module, etc. The detailed meanings of the symbols can refer to ISO 6336-3. The meanings of the symbols and an example of RBS calculating for the paired gear rings on torque 100 Nm are recorded in Table 1. Here, the values of \( K_A, K_F, K_{F_0}, K_{F_{\beta}} \) are set to 1 for the ideal gears since they are very difficult to determine exactly.

<table>
<thead>
<tr>
<th>Gear rings</th>
<th>Inner</th>
<th>Outer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal tangential load (N)</td>
<td>( F_t )</td>
<td>4167</td>
</tr>
<tr>
<td>Tooth number (Z)</td>
<td>( Z )</td>
<td>48</td>
</tr>
<tr>
<td>Face width (mm)</td>
<td>( b )</td>
<td>10</td>
</tr>
<tr>
<td>Normal module (mm)</td>
<td>( m_n )</td>
<td>1</td>
</tr>
<tr>
<td>Form factor</td>
<td>( Y_F )</td>
<td>1.51</td>
</tr>
<tr>
<td>Stress correction factor</td>
<td>( Y_S )</td>
<td>1.2</td>
</tr>
<tr>
<td>Deep tooth factor</td>
<td>( Y_{DT} )</td>
<td>0.89</td>
</tr>
<tr>
<td>Helix angle factor</td>
<td>( Y_{\beta} )</td>
<td>1</td>
</tr>
<tr>
<td>Rim thickness factor</td>
<td>( Y_{R} )</td>
<td>1</td>
</tr>
<tr>
<td>Application factor</td>
<td>( K_A )</td>
<td>1</td>
</tr>
<tr>
<td>Dynamic factor</td>
<td>( K_F )</td>
<td>1</td>
</tr>
<tr>
<td>Face load factor</td>
<td>( K_{F_0} )</td>
<td>1</td>
</tr>
<tr>
<td>Transverse load factor</td>
<td>( K_{F_{\beta}} )</td>
<td>1</td>
</tr>
<tr>
<td>Root Bending Stress (MPa)</td>
<td>( \sigma_v )</td>
<td>671.95</td>
</tr>
</tbody>
</table>

2.2 Contact stresses

Pitting is a surface fatigue failure due to many repetition of high contact stresses occurring in the gear tooth surface while a pair of teeth is transmitting power. Contact stress of gears is commonly predicted based on Hertz theory which the contact points of the gear teeth are simulated with two contacted cylinders. According to Hertz theory as given in, when two cylinders are pressed together, the surface contact stress (SCS) is given by,

\[ \sigma_H = \frac{2F}{\pi BL}, \]  

and \( B = \sqrt{\frac{2F\left(1 - \frac{\mu_a^2}{E_1} + \frac{1 - \mu_b^2}{E_2}\right)}{\pi d \left(\frac{1}{d_1} + \frac{1}{d_2}\right)}} \)

Where,
σ_H: The maximum value of contact stress (N/mm²)
F: The normal contact force pressing the two cylinders together (N)
B: The half width of deformation (mm)
L: The axial length of cylinders (mm)
d1,d2: The diameters of two cylinders (mm)
E1,E2: The modules of elasticity of two cylinder materials (N/mm²)
μ1, μ2: The poison’s ratio of the two cylinder materials

Moreover, the ISO standards 6336-2 also introduced the methods to calculate the contact stress for a pair of spur gears and helical gears[8]. The formula defined in ISO 6336-2 for the SCS calculation of a pair of spur gears and helical gears having contact ratios in the range 1<c<2 is defined as

\[ \sigma_H = Z_d Z_u Z_c Z_H \frac{F_t}{b d} \left( \frac{u \pm 1}{u} \right) K_c K_K K_{H1} \]  

(4)

where the "+" symbol in equations applies to two external gears in mesh, and the "−" symbol is used for an internal gear and an external gear mesh. The values of these factors can be determined according to the technical data reported in Ref.[9]. The meanings of the symbols and an example of the SCS for the paired gear rings are listed in Table 2.

Table 2. Surface contact stresses of the paired gears

<table>
<thead>
<tr>
<th>Gear rings</th>
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<th>Outer</th>
</tr>
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<tbody>
<tr>
<td>Nominal tangential load (N)</td>
<td>( F_t )</td>
<td>4167</td>
</tr>
<tr>
<td>Tooth number (Z)</td>
<td>( Z )</td>
<td>48</td>
</tr>
<tr>
<td>Reference diameter (mm)</td>
<td>( d_1 )</td>
<td>48</td>
</tr>
<tr>
<td>Face width (mm)</td>
<td>( b )</td>
<td>10</td>
</tr>
<tr>
<td>Gear ratio (Z2/Z1)</td>
<td>( u )</td>
<td>1.04</td>
</tr>
<tr>
<td>Single pair tooth contact factor of the wheel</td>
<td>( Z_D )</td>
<td>1</td>
</tr>
<tr>
<td>Zone factor</td>
<td>( Z_H )</td>
<td>2.5</td>
</tr>
<tr>
<td>Elasticity factor</td>
<td>( Z_E )</td>
<td>186.96</td>
</tr>
<tr>
<td>Contact ratio factor</td>
<td>( Z_C )</td>
<td>0.97</td>
</tr>
<tr>
<td>Helix angle factor</td>
<td>( Z_H )</td>
<td>1</td>
</tr>
<tr>
<td>Application factor</td>
<td>( K_A )</td>
<td>1</td>
</tr>
<tr>
<td>Dynamic factor</td>
<td>( K_V )</td>
<td>1</td>
</tr>
<tr>
<td>Face load factor</td>
<td>( K_{H1} )</td>
<td>1</td>
</tr>
<tr>
<td>Transverse load factor</td>
<td>( K_{H2} )</td>
<td>1</td>
</tr>
<tr>
<td>Surface contact stress (MPa)</td>
<td>( \sigma_H )</td>
<td>1870.82</td>
</tr>
</tbody>
</table>

4. Structural stress analysis

Structural stress of the gear rings is analyzed using finite element methods. The analyzed results are used to evaluate the fatigue life of the gear rings so that the expected life of the DR under working loadings be given. The geometric models of the inner-outer gear rings were leaded into ANSYS to perform structural stress analysis. The bending and contact stresses generated at the mating teeth are the major concerned problems. The analyzed results are compared with the formula calculations to identify the accuracy of analyzing. The material of the gear rings are set to structural steel.

The meshing and the analyzed setting for the paired gear rings are shown in Figure 2. The meshes are strengthened at the mating areas of the teeth for obtaining fine solutions of analyzing. The analyzed results for the equivalent stresses as well as the SCS are shown in Figure 3. The maximum stress is 1788.8 MPa occurring at the contacting points of the engaging teeth.
Figure 2. The setting of analyzing: (a) Meshing, (b) The supports and the loadings

Figure 3. Surface contacting stresses

Figure 4. Tooth root bending stresses

On the other hand, the analyzed results of the principle stresses of the gear rings as well as its RBS are shown in Figure 4. The maximum stresses is 600 MPa which occurs at the roots of teeth. The analyzed results of stresses are similar to the values of formula calculations. The results imply that the FEA methods can replace the formula calculations to obtain the contacting and bending stresses of the gear rings since the two methods only exist a small evaluated error. It is convenient to evaluating the induced stresses of the gear rings by FTA since the formula calculations involve many complex procedures in determining the related parameters.
5. Fatigue life estimation

The fatigue resistance as well as the strength of the gear rings often is demanded so that the DR has adequate strength and a high number of loading cycles. The formulas to evaluate the permissible strength for the volumetric and superficial fatigue of steel with application on cylindrical involute gears with external teeth gears are given in AGMA Standard 2001-D04[9]. The stresses are defined as

\[
\sigma_f = \frac{\sigma_{F,\text{lim}} \cdot Y_N}{K_F \cdot Y_F \cdot Y_T} \quad (5)
\]

\[
\sigma_H = \frac{\sigma_{H,\text{lim}} \cdot Z_N \cdot Z_W}{K_H \cdot Y_H \cdot Y_Z}
\]

\(\sigma_f\): Permissible bending stress taking into account fatigue strength, [MPa].  
\(\sigma_H\): Permissible contact stress taking into account fatigue strength, [MPa].  
\(\sigma_{F,\text{lim}}\): Fatigue limit for bending stress and unidirectional loading, [MPa].  
\(\sigma_{H,\text{lim}}\): Fatigue limit taking into account contact stress, [MPa].  
\(K_F\): Safety factor for bending strength.  
\(K_H\): Safety factor for pitting.  
\(Y_N\): Stress cycle factor for bending strength.  
\(Z_N\): Stress cycle factor for pitting resistance.  
\(Y_F\): Temperature factor.  
\(Y_W\): Reliability factor.  
\(Z_W\): Hardness ratio factor for pitting resistance.

The actual cylindrical gear-tooth rating formula for bending stresses is based on cantilever projection theory. The maximum tensile stress at the tooth-root (in the direction of the tooth height) which may not exceed the permissible bending stress for the material is the basis for rating the bending strength of gear teeth. They have also been improved with modifications in the new standards to consider load sharing between adjacent teeth, the load increment due to external and internal dynamic loads, uneven distribution of load over the face-width due to mesh misalignment caused by inaccuracies in manufacture, and elastic deformations, etc.

The procedure and formulas to estimate the gear lives for a high number of cycles has been suggested in AGMA Standard 2001-D04. The fatigue life functions of bending stress for different material harnesses are expressed as a function of stress cycle factor \(Y_N\). The function, for example, for material hardness HB=260, is about as

\[
Y_N = 6.1514N^{-0.1192}
\]

On other hand, the fatigue lives for contact stress is formulated with a pitting resistance stress factor, \(Z_N\), as

\[
Z_N = 2.466N^{-0.106}
\]

According to the life equations, the fatigue lives of gear for bending and contact stresses can be evaluated while the stress cycle factors \(Y_N\) and \(Z_N\) are given, respectively. The stress cycle factors are determined depending on the mechanical characteristics of gear material such as yielding strength, surface hardness and corrosion resistance, and the operational environments such as the temperature and lubrication, etc.

The stress cycle factors \(Z_N\) and \(Y_N\) can be determined according to the permissible stresses obtained by FEA. No sooner are the RBS and the SCS of the gear rings determined than the fatigue lives can be estimated by the corresponding stress cycle factors. For example, the gear ring designed with structural alloy steel SCM415and is harden processing to surface hardness Hv=600, core hardness HB=260.The fatigue limits for bending and contacting are \((\sigma_{F,\text{lim}},\sigma_{H,\text{lim}})=(42.5, 164)\) kgf/mm², respectively. The evaluated values of the stress cycle factors and the fatigue lives are listed in Table 3. The result show that the bending fatigue life is shorter than the contacting fatigue life. It means that the expected lives of the gear rings as well as the DR is dominated
6. Conclusions

This paper presents a novel design of high speed-reduced ratio’s reducers based on differential displacements of the paired gear rings. The teeth of the gear rings are designed with profile-shafted involute to avoid interference occurrence of the paired gears. The induced stresses of the gear rings including tooth root bending and surface contacting are reported for provide the analytical solutions of the mechanism strengths. The estimated models of fatigue lifetime of gears are proposed for predicting the expected lives of the reducers. FEM is used to analyze the induced stresses of the gear rings under different loadings. The analyzed results are used to evaluate the expected lives in cooperation with the life estimation models. In this paper, an effective procedure, formulas, and information to estimate the expected fatigue life of the paired gears are given based on the combination of the analytical models and FEM. The formulas are developed based on the AGMA Standard for calculating the allowed loadings of the reducers. Some results of field studies show a good approximation between data from the field and the values obtained by means of the procedure described in this paper. It is necessary to conduct more testing and data application to improve the results due to many factors need to be considered.

Acknowledgements

The work was supported by a grant from the National Science Council under contract No: MOST 104-2221-E-237-001. The authors would like to appreciate the reviewers for their valuable suggestions.

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